

Seria Fourier în timp continuu

$$x(t) = \sum_{k=-\infty}^{\infty} c_k^x e^{jk\omega_0 t} \longrightarrow c_k^x = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt, \quad \omega_0 = 2\pi/T$$

Tabelul 4.1 Proprietățile seriei Fourier exponențiale

Semnal periodic	Coefficienții seriei Fourier exponențiale
$x(t) = x(t+T), \quad \omega_0 = 2\pi/T$	$\{c_k^x\}$
$y(t) = y(t+T), \quad \omega_0 = 2\pi/T$	$\{c_k^y\}$
$ax(t) + by(t)$	$\{ac_k^x + bc_k^y\}$
$x(t-t_0), \quad t_0 \in \mathbb{R}$	$\{e^{-jk\omega_0 t_0} c_k^x\} \quad \omega_0 = 2\pi/T$
$x(t)e^{jk_0\omega_0 t}, \quad k_0 \in \mathbb{Z}, \quad \omega_0 = 2\pi/T$	$\{c_{k-k_0}^x\}$
$x^*(t)$	$\{(c_{-k}^x)^*\}$
$x(-t)$	$\{c_{-k}^x\}$
$x(at), \quad a \in \mathbb{R}_+^*, \quad \text{perioada } T/a$	$\{c_k^x\}$
$x(t) \otimes y(t) = \int_T x(\tau) y(t-\tau) d\tau$	$\{Tc_k^x c_k^y\}$
$\frac{1}{T} \int_T x^*(\tau) y(t+\tau) d\tau$	$\{(c_k^x)^* c_k^y\}$
$x(t)y(t)$	$\{c_k^x * c_k^y\} = \left\{ \sum_{n=-\infty}^{\infty} c_{k-n}^x c_n^y \right\}$
$\frac{dx(t)}{dt}$	$\{jk\omega_0 c_k^x\}$
$\int_{-\infty}^t x(\tau) d\tau$	$\left\{ \frac{c_k^x}{jk\omega_0} \right\} \quad c_0^x = 0$
$x(t) \in \mathbb{R}$	$c_k^x = (c_{-k}^x)^*$ $ c_k^x = c_{-k}^x ; \quad \text{Arg } c_k^x = -\text{Arg } c_{-k}^x$ $\text{Re}\{c_k^x\} = \text{Re}\{c_{-k}^x\}; \quad \text{Im}\{c_k^x\} = -\text{Im}\{c_{-k}^x\}$
$x_p(t), \quad x(t) \in \mathbb{R}$	$\{\text{Re } c_k^x\}$
$x_i(t), \quad x(t) \in \mathbb{R}$	$\{j \text{Im } c_k^x\}$
$\frac{1}{T} \int_T x(t) ^2 dt = \sum_{k=-\infty}^{\infty} c_k ^2$	

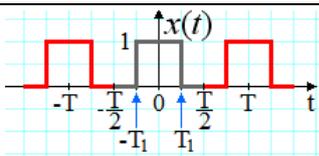
Transformarea Fourier în timp continuu

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \longrightarrow X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Tabelul 5.3 Proprietățile transformării Fourier în timp continuu

Semnalul aperiodic	Transformata Fourier
$x(t)$	$X(\omega)$
$y(t)$	$Y(\omega)$
$ax(t) + by(t)$	$aX(\omega) + bY(\omega)$
$x(t - t_0), t_0 \in \mathbb{R}$	$e^{-j\omega t_0} X(\omega)$
$e^{j\omega_0 t} x(t), \omega_0 \in \mathbb{R}$	$X(\omega - \omega_0)$
$x^*(t)$	$X^*(-\omega)$
$x(-t) = \check{x}(t)$	$X(-\omega)$
$x(at), a \in \mathbb{R}$	$\frac{1}{ a } X\left(\frac{\omega}{a}\right)$
$x(t) * y(t)$	$X(\omega) \cdot Y(\omega)$
$\check{x}^*(t) * y(t)$	$X^*(\omega) \cdot Y(\omega)$
$x(t)y(t)$	$\frac{1}{2\pi} X(\omega) * Y(\omega)$
$\frac{dx(t)}{dt}$	$j\omega X(\omega)$
$\int_{-\infty}^t x(\tau) d\tau$	$\frac{1}{j\omega} X(\omega) + \pi X(0)\delta(\omega)$
$tx(t)$	$j \frac{d}{d\omega} X(\omega)$
$x(t) \in \mathbb{R}$	$X(\omega) = X^*(-\omega)$ $ X(\omega) = X(-\omega) ; \quad \text{Arg } X(\omega) = -\text{Arg } X(-\omega)$ $\text{Re}\{X(\omega)\} = \text{Re}\{X(-\omega)\}; \quad \text{Im}\{X(\omega)\} = -\text{Im}\{X(-\omega)\}$
$x_p(t), x(t) \in \mathbb{R}$	$\text{Re}\{X(\omega)\}$
$x_i(t), x(t) \in \mathbb{R}$	$j \text{Im}\{X(\omega)\}$
$\int_{-\infty}^{\infty} x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) ^2 d\omega$	
$x(t) \longleftrightarrow X(\omega)$	
$X(t) \longleftrightarrow 2\pi x(-\omega)$	

Tabelul 5.4 Perechi semnal – transformata/serie Fourier

Semnalul	Transformata Fourier	Coefficienții seriei Fourier exponențiale (doar pentru semnale periodice)
$\sum_k c_k e^{jk\omega_0 t}$	$\sum_k 2\pi c_k \delta(\omega - k\omega_0)$	$\{c_k\}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	$c_1 = 1; c_k = 0, k \neq 1$
$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$c_1 = c_{-1} = \frac{1}{2};$ $c_k = 0, k \notin \{-1, 1\}$
$\sin \omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$c_1 = -c_{-1} = \frac{1}{2j};$ $c_k = 0, k \notin \{-1, 1\}$
$x(t) = 1(\text{const})$	$2\pi\delta(\omega)$	$c_0 = 1; c_k = 0, k \neq 0$
	$\sum_{k=-\infty}^{\infty} \frac{2 \sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$	$c_k = \frac{\sin k\omega_0 T_1}{k\pi}, k \neq 0;$ $c_0 = \frac{2T_1}{T}$
$\delta_T(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$	$\omega_0 \delta_{\omega_0}(t) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega - k \frac{2\pi}{T}\right)$	$c_k = \frac{1}{T}$
$p_\tau(t) = \begin{cases} 1, & t < \tau \\ 0, & t > \tau \end{cases}$	$\frac{2 \sin \omega\tau}{\omega}$	—
$\frac{\sin \omega_0 t}{\pi t}$	$p_{\omega_0}(\omega) = \begin{cases} 1, & \omega < \omega_0 \\ 0, & \omega > \omega_0 \end{cases}$	—
$\delta(t)$	$1(\text{const})$	—
$\delta(t - t_0), t_0 \in \mathbb{R}$	$e^{-j\omega t_0}$	—
$\sigma(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$	—
$e^{-at} \sigma(t), \text{Re}\{a\} > 0$	$(a + j\omega)^{-1}$	—
$t e^{-at} \sigma(t), \text{Re}\{a\} > 0$	$(a + j\omega)^{-2}$	—
$\frac{t^{n-1} e^{-at}}{(n-1)!} \sigma(t), \text{Re}\{a\} > 0$	$(a + j\omega)^{-n}$	—

Seria Fourier în timp discret

$$x[n] = \sum_{k=0}^{N-1} c_k e^{jk\Omega_0 n}, n = \overline{0, N-1} \longrightarrow c_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\Omega_0 n}, k = \overline{0, N-1}, \quad \Omega_0 = \frac{2\pi}{N}$$

Tabelul 6.1 Proprietățile seriei Fourier în timp discret

Semnalul discret periodic	Coefficienții seriei Fourier exponențiale
$x[n], x[n+N] = x[n] = x[(n)_N]$	$\{c_k^x\}; c_{k+N}^x = c_k^x = c_{(k)_N}^x$
$y[n], y[n+N] = y[n] = y[(n)_N]$	$\{c_k^y\}; c_{k+N}^y = c_k^y = c_{(k)_N}^y$
$ax[n] + by[n]$	$\{ac_k^x + bc_k^y\}$
$x[n - n_0], n_0 \in \mathbb{Z}$	$\{e^{-jk\frac{2\pi}{N}n_0} c_k^x\}$
$x^*[n]$	$\{(c_{-k}^x)^*\}$
$x[-n]$	$\{c_{-k}^x\}$
$x_{(m)}[n] = \begin{cases} x[n/m]; & \text{daca } n:m \\ 0 & ; \text{ in rest} \end{cases}, \text{ perioada } mN$	$\left\{\frac{1}{m} c_k^x\right\}, \text{ perioada } mN$
$e^{jk_0\frac{2\pi}{N}n} x[n]$	$\{c_{k-k_0}^x\}$
$x[n] \otimes y[n] = \sum_{k \in (N)} x[k] y[n-k]$	$\{N c_k^x c_k^y\}$
$\frac{1}{N_0} \sum_{k=0}^{N_0} x^*[n] y[n+k]$	$\{(c_k^x)^* \cdot c_k^y\}$
$x[n] y[n]$	$\{c_k^x \otimes c_k^y\} = \left\{ \sum_{m \in (N)} c_{k-m}^x c_m^y \right\}$
$x[n] - x[n-1]$	$\{(1 - e^{-j2\pi k/N}) c_k^x\}$
$\sum_{k=-\infty}^n x[k], c_0^x = 0$	$\{c_k^x / (1 - e^{-j2\pi k/N})\}, c_0^x = 0$
$x[n] \in \mathbb{R}$	$c_k = c_{-k}^* = c_{(-k)_N}^*$ $ c_k = c_{-k} ; \quad \text{Arg } c_k = -\text{Arg } c_{-k}$ $\text{Re}\{c_k\} = \text{Re}\{c_{-k}\}; \quad \text{Im}\{c_k\} = -\text{Im}\{c_{-k}\}$
$x_p[n], x[n] \in \mathbb{R}$	$\{\text{Re}\{c_k^x\}\}$
$x_i[n], x[n] \in \mathbb{R}$	$\{j \text{Im}\{c_k^x\}\}$
$\frac{1}{N} \sum_{n \in (N)} x[n] ^2 = \sum_{k \in (N)} c_k ^2$	

Transformata Fourier în timp discret

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(\Omega) e^{j\Omega n} d\Omega \longrightarrow X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$$

Tabelul 6.2 Proprietățile transformării Fourier în timp discret

Semnalul discret aperiodic	Transformata Fourier
$x[n]$	$X(\Omega); X(\Omega + 2\pi) = X(\Omega)$
$y[n]$	$Y(\Omega); Y(\Omega + 2\pi) = Y(\Omega)$
$ax[n] + by[n]$	$aX(\Omega) + bY(\Omega)$
$x[n - n_0]$	$e^{-j\Omega n_0} X(\Omega)$
$e^{j\Omega_0 n} x[n]$	$X(\Omega - \Omega_0)$
$x^*[n]$	$X^*(-\Omega)$
$x[-n]$	$X(-\Omega)$
$x_{(k)}[n] = \begin{cases} x[n/k]; & \text{daca } n:k \\ 0 & ; \text{ in rest} \end{cases}$	$X(k\Omega)$
$x[n] * y[n]$	$X(\Omega)Y(\Omega)$
$\tilde{x}^*[n] * y[n]$	$X^*(\Omega)Y(\Omega)$
$x[n]y[n]$	$\frac{1}{2\pi} X(\Omega) \otimes Y(\Omega) = \frac{1}{2\pi} \int_{2\pi} X(u)Y(\Omega - u) du$
$x[n] - x[n-1]$	$(1 - e^{-j\Omega}) X(\Omega)$
$\sum_{k=-\infty}^n x[k]$	$\frac{X(\Omega)}{1 - e^{-j\Omega}} + \pi X(0) \sum_{k=-\infty}^{\infty} \delta(\Omega - k2\pi)$
$nx[n]$	$j \frac{dX(\Omega)}{d\Omega}$
$x[n] \in \mathbb{R}$	$X(\Omega) = X^*(-\Omega);$ $ X(\Omega) = X(-\Omega) ; \quad \text{Arg } X(\Omega) = -\text{Arg } X(-\Omega)$ $\text{Re}\{X(\Omega)\} = \text{Re}\{X(-\Omega)\}; \quad \text{Im}\{X(\Omega)\} = -\text{Im}\{X(-\Omega)\}$
$x_p[n], x[n] \in \mathbb{R}$	$\text{Re}\{X(\Omega)\}$
$x_i[n], x[n] \in \mathbb{R}$	$j\text{Im}\{X(\Omega)\}$
$\sum_{n=-\infty}^{\infty} x[n] ^2 = \frac{1}{2\pi} \int_{2\pi} X(\Omega) ^2 d\Omega$	

Tabelul 6.3 Perechi semnal-transformata Fourier în timp discret/serie Fourier în timp discret

Semnalul	Transformata Fourier	Coefficienții seriei Fourier exponențiale (doar pentru semnale periodice)
$\sum_{k \in \langle N \rangle} c_k e^{jk \frac{2\pi}{N} n}$	$\sum_{k \in \langle N \rangle} 2\pi c_k \delta(\Omega - k2\pi/N); \quad c_k = c_{(k)_N}$	$\{c_k\}$
$e^{j\Omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi \delta(\Omega - \Omega_0 - k2\pi)$	daca $\frac{\Omega_0}{2\pi} = \frac{m}{N} \in \mathbb{Q}: \{c_k\}; c_k = \begin{cases} 1, & (k)_N = m \\ 0, & \text{in rest} \end{cases}$ daca $\frac{\Omega_0}{2\pi} \notin \mathbb{Q}$: semnalul nu este periodic
$\cos \Omega_0 n$	$\sum_{k=-\infty}^{\infty} \pi [\delta(\Omega - \Omega_0 - k2\pi) + \delta(\Omega + \Omega_0 - k2\pi)]$	daca $\frac{\Omega_0}{2\pi} = \frac{m}{N} \in \mathbb{Q}: \{c_k\}, c_k = c_{-k} = \begin{cases} 1/2, & (k)_N = m \\ 0, & \text{in rest} \end{cases}$ daca $\frac{\Omega_0}{2\pi} \notin \mathbb{Q}$: semnalul nu este periodic
$\sin \Omega_0 n$	$\sum_{k=-\infty}^{\infty} \frac{\pi}{j} [\delta(\Omega - \Omega_0 - k2\pi) - \delta(\Omega + \Omega_0 - k2\pi)]$	daca $\frac{\Omega_0}{2\pi} = \frac{m}{N} \in \mathbb{Q}: c_k = c_{-k} = \begin{cases} \frac{1}{2j}, & k = m \pm pN \\ -\frac{1}{2j}, & k = -m \pm pN \\ 0, & \text{in rest} \end{cases}$ daca $\frac{\Omega_0}{2\pi} \notin \mathbb{Q}$: semnalul nu este periodic
$x[n] = 1$	$\sum_{k=-\infty}^{\infty} 2\pi \delta(\Omega - k2\pi)$	$c_k = \begin{cases} 1, & (k)_N = m \\ 0, & \text{in rest} \end{cases}$
$x[n] = \begin{cases} 1, & n \leq N_1 \\ 0, & N_1 \leq n \leq \frac{N}{2} \end{cases}$ $x[n+N] = x[n]$	$\sum_{k=-\infty}^{\infty} 2\pi \frac{\sin\left(\frac{2N_1+1}{2} \cdot \frac{2k\pi}{N}\right)}{N \sin \frac{2k\pi}{2N}} \cdot \delta\left(\Omega - k \frac{2\pi}{N}\right)$	$c_k = \frac{\sin\left(\frac{2N_1+1}{2} \cdot \frac{2k\pi}{N}\right)}{N \sin \frac{2k\pi}{2N}}, (k)_N \neq 0;$ $c_k = \frac{2N_1+1}{2}, (k)_N = 0$

Semnalul	Transformata Fourier	Coefficienții seriei Fourier exponențiale (doar pentru semnale periodice)
$\delta_N[n] = \sum_{k=-\infty}^{\infty} \delta[n-kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\Omega - k \frac{2\pi}{N}\right)$	$c_k = \frac{1}{N}$
$a^n \sigma[n], a < 1$	$\frac{1}{1 - ae^{-j\Omega}}$	-
$x[n] = \begin{cases} 1, & n \leq N_1 \\ 0, & n > N_1 \end{cases}$	$\frac{\sin[(2N_1 + 1)/2]\Omega}{\sin(\Omega/2)}$	-
$\frac{\sin \Omega_0 n}{\pi n}; 0 < \Omega_0 < \pi$	$X(\Omega) = \begin{cases} 1, & \Omega \leq \Omega_0 \\ 0, & \Omega_0 < \Omega \leq \pi \end{cases}$ $X(\Omega + 2\pi) = X(\Omega)$	-
$\delta[n]$	1 (constant)	-
$\sigma[n]$	$\frac{1}{1 - e^{-j\Omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\Omega - 2k\pi)$	-
$\delta[n - n_0]$	$e^{-j\Omega n_0}$	-
$(n+1)a^n \sigma[n];$ $ a < 1$	$\frac{1}{(1 - ae^{-j\Omega})^2}$	-
$\frac{(n+m-1)!}{n!(m-1)!} a^n \sigma[n];$ $ a < 1$	$\frac{1}{(1 - ae^{-j\Omega})^m}$	-

Transformata Laplace

$$x(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st} ds, \operatorname{Re}\{s\} \in \text{ROC} \longrightarrow X(s) = \int_{-\infty}^{\infty} x(t)e^{-st} dt; \quad (s = \sigma + j\omega) - \text{transformata Laplace bilaterală}$$

$$x(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st} ds, \operatorname{Re}\{s\} > \sigma_0 \longrightarrow X_u(s) = \int_{0^+}^{\infty} x(t)e^{-st} dt; \quad (s = \sigma + j\omega) - \text{transformata Laplace unilaterială}$$

Tabelul 7.1 Proprietățile transformării Laplace

Semnalul	Transformarea bilaterală		Transformarea unilaterială
	Transformata	Domeniul de convergență	
$x(t)$	$X(s)$	DC^x	$X_u(s)$
$y(t)$	$Y(s)$	DC^y	$Y_u(s)$
$ax(t) + by(t)$	$aX(s) + bY(s)$	$DC^x \cap DC^y$ cel puțin	$aX_u(s) + bY_u(s)$
$x(t - t_0), t_0 \in \mathbb{R}$	$e^{-st_0} X(s)$	DC^x	—
$x(t - t_0), t_0 > 0$	$e^{-st_0} X(s)$	DC^x	$e^{-st_0} X_u(s)$
$e^{s_0 t} x(t)$	$X(s - s_0)$	DC^x deplasat	$X_u(s - s_0)$
$x(at), a \in \mathbb{R}^*$	$\frac{1}{ a } X\left(\frac{s}{a}\right)$	DC^x scalat	—
$x(at), a > 0$	$\frac{1}{a} X\left(\frac{s}{a}\right)$	DC^x scalat	$\frac{1}{a} X_u\left(\frac{s}{a}\right)$
$x(t) * y(t)$	$X(s)Y(s)$	$DC^x \cap DC^y$ cel puțin	$X_u(s)Y_u(s)$
$\frac{d}{dt} x(t)$	$sX(s)$	DC^x cel puțin	$sX_u(s) - x(0^+)$

Semnalul	Transformarea bilaterală		Transformarea unilaterală
	Transformata	Domeniul de convergență	
$-tx(t)$	$\frac{d}{ds} X(s)$	DC^x	$\frac{d}{ds} X_u(s)$
$(-t)^n x(t)$	$\frac{d^n}{ds^n} X(s)$	DC^x	$\frac{d^n}{ds^n} X_u(s)$
$x(t)y(t)$	$\frac{1}{2\pi j} \oint_{\Gamma} X(u)Y(s-u)du$	$\Gamma \subset DC^x \cap DC^y$ cel puțin	$\frac{1}{2\pi j} \oint_{\Gamma} X(u)Y(s-u)du$
$\int_{-\infty}^t x(\tau)d\tau$	$\frac{1}{s} X(s)$	$DC^x \cap \{\text{Re}\{s\} > 0\}$ cel puțin	—
$\int_0^t x(\tau)d\tau$	$\frac{1}{s} X(s)$	$DC^x \cap \{\text{Re}\{s\} > 0\}$ cel puțin	$\frac{X_u(s) + x^{(-1)}(0)}{s}$ $x^{(-1)}(0)$ – valoarea initiala a integralei
$x(t) = x(t)\sigma(t)$	$x(0^+) = \lim_{s \rightarrow \infty} sX(s)$		$x(0^+) = \lim_{s \rightarrow \infty} sX_u(s)$
$x(t) = x(t)\sigma(t)$	$x(\infty) = \lim_{s \rightarrow 0} sX(s)$		$x(\infty) = \lim_{s \rightarrow 0} sX_u(s)$

Tabelul 7.2 Perechi semnal-transformata Laplace

Semnalul	Transformata	Domeniul de convergență
$\delta(t)$	1 (constant)	$\forall s$
$\sigma(t)$	$\frac{1}{s}$	$\operatorname{Re}\{s\} > 0$
$^* -\sigma(-t)$	$\frac{1}{s}$	$\operatorname{Re}\{s\} < 0$
$\frac{t^{n-1}}{(n-1)!} \sigma(t)$	$\frac{1}{s^n}$	$\operatorname{Re}\{s\} > 0$
$^* -\frac{t^{n-1}}{(n-1)!} \sigma(-t)$	$\frac{1}{s^n}$	$\operatorname{Re}\{s\} < 0$
$e^{-at} \sigma(t)$	$\frac{1}{s+a}$	$\operatorname{Re}\{s\} > -a$
$^* -e^{-at} \sigma(-t)$	$\frac{1}{s+a}$	$\operatorname{Re}\{s\} < -a$
$\frac{t^{n-1}}{(n-1)!} e^{-at} \sigma(t)$	$\frac{1}{(s+a)^n}$	$\operatorname{Re}\{s\} > -a$
$^* -\frac{t^{n-1}}{(n-1)!} e^{-at} \sigma(-t)$	$\frac{1}{(s+a)^n}$	$\operatorname{Re}\{s\} < -a$
$\delta(t-t_0); t_0 > 0$	e^{-st_0}	$\forall s$
$(\cos \omega_0 t) \sigma(t)$	$\frac{s}{s^2 + \omega_0^2}$	$\operatorname{Re}\{s\} > 0$
$(\sin \omega_0 t) \sigma(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$	$\operatorname{Re}\{s\} > 0$
$(e^{-at} \cos \omega_0 t) \sigma(t)$	$\frac{s+a}{(s+a)^2 + \omega_0^2}$	$\operatorname{Re}\{s\} > -a$
$(e^{-at} \sin \omega_0 t) \sigma(t)$	$\frac{\omega_0}{(s+a)^2 + \omega_0^2}$	$\operatorname{Re}\{s\} > -a$
$J_0(at) \sigma(t)$	$\frac{1}{\sqrt{s^2 + a^2}}$	$\operatorname{Re}\{s\} > - a $

Pentru semnalele cauzale, $X(s) = X_u(s)$, iar pentru cele necauzale există numai $X(s)$.

Transformata Z

$$x[n] = \frac{1}{2\pi j} \oint_{\Gamma} X(z) z^{n-1} dz, \quad \Gamma \subset ROC \longrightarrow X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} - \text{transformata Z bilaterală,}$$

$$z = r \cdot e^{j\Omega}, \quad r \geq 0$$

$$x[n] \longrightarrow X_u(z) = \sum_{n=0}^{\infty} x[n] z^{-n} - \text{transformata Z unilaterială}$$

Tabelul 10.1 Proprietățile transformării Z

Semnalele marcate cu * corespund numai transformării unilaterale. În rest, proprietățile celor două transformate sunt identice.

Semnalul	Transformata Z	Domeniul de convergență
$x[n]$	$X(z)$	$R_-^x < z < R_+^x$
$y[n]$	$Y(z)$	$R_-^y < z < R_+^y$
$ax[n] + by[n]$	$aX(z) + bY(z)$	$\max\{R_-^x, R_-^y\} < z < \min\{R_+^x, R_+^y\}$
$x[n - n_0], n_0 \in \mathbb{Z}$	$z^{-n_0} X(z)$	$R_-^x < z < R_+^x$
* $x[n - n_0], n_0 \in \mathbb{N}$	$z^{-n_0} \left(X(z) + \sum_{n=-n_0}^{-1} x[n] z^{-n} \right)$	$R_-^x < z $
* $x[n + n_0], n_0 \in \mathbb{N}$	$z^{n_0} \left(X(z) - \sum_{n=0}^{n_0-1} x[n] z^{-n} \right)$	$R_-^x < z $
$e^{j\Omega_0 n} x[n]$	$X(e^{-j\Omega_0} z)$	$R_-^x < z < R_+^x$
$x[-n]$	$X(1/z)$ numai pentru transformarea bilaterală	$\frac{1}{R_+^x} < z < \frac{1}{R_-^x}$
$x[n] - x[n-1]$	$(1 - z^{-1}) X(z)$	$R_-^x < z < R_+^x$
* $x[n] - x[n-1]$	$(1 - z^{-1}) X(z) - x[-1]$	$R_-^x < z $
$\sum_{k=-\infty}^n x[k]$	$\frac{X(z)}{1 - z^{-1}}$	$R_-^x < z < R_+^x$
* $\sum_{k=-\infty}^n x[k]$	$\frac{X(z) + \sum_{k=-\infty}^{-1} x[k]}{1 - z^{-1}}$	$R_-^x < z $
$nx[n]$	$-z \frac{d}{dz} X(z)$	$R_-^x < z < R_+^x$
$x[n]; x[n] \equiv 0 \ n < 0$	$\lim_{z \rightarrow \infty} X(z) = x[0]$	$R_-^x < z $
$x[n]; x[n] \equiv 0 \ n < 0$	$\lim_{z \rightarrow 1} (z-1) X(z) = x[\infty]$	$R_-^x < z $
$x^*[n]$	$X^*(z^*)$	$R_-^x < z < R_+^x$

Semnalul	Transformata Z	Domeniul de convergență
$x[n] * y[n]$	$X(z)Y(z)$	$\max\{R_-^x, R_-^y\} < z $ $ z < \min\{R_+^x, R_+^y\}$
$x[n]y[n]$	$\frac{1}{2\pi j} \oint_{\Gamma} X(u)Y\left(\frac{z}{u}\right)\frac{du}{u}$	$R_-^x R_-^y < z < R_+^x R_+^y$
$\sum_{n=-\infty}^{\infty} x[n] ^2 = \frac{1}{2\pi j} \oint_{\Gamma} X(u)X^*(1/u^*)\frac{du}{u}$		

Tabelul 10.2 Perechile semnal-transformata Z

Cu excepția semnalelor marcate în tabel cu * ce nu au transformată unilaterală, cele 2 transformate (unilaterală și bilaterală) sunt identice.

Semnalul	Transformata Z	Domeniul de convergență
$\delta[n]$	1 (constant)	$\forall z \in \mathbb{C}$
$\sigma[n]$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$	$ z > 1$
* $-\sigma[-n-1]$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$	$ z < 1$
$\delta[n-n_0]$	z^{-n_0}	$\forall z \in \mathbb{C} \setminus \{0\}$ dacă $n_0 > 0$ sau $\forall z \in \mathbb{C} \setminus \{\infty\}$ dacă $n_0 < 0$
$a^n \sigma[n]$	$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$	$ z > a $
* $-a^n \sigma[-n-1]$	$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$	$ z < a $
$na^n \sigma[n]$	$\frac{az^{-1}}{(1-az^{-1})^2} = \frac{az}{(z-a)^2}$	$ z > a $
* $-na^n \sigma[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2} = \frac{az}{(z-a)^2}$	$ z < a $
$\sigma[n] \cos \Omega_0 n$	$\frac{1-z^{-1} \cos \Omega_0}{1-2z^{-1} \cos \Omega_0 + z^{-2}} = \frac{z(z-\cos \Omega_0)}{z^2-2z \cos \Omega_0 + 1}$	$ z > 1$
$\sigma[n] \sin \Omega_0 n$	$\frac{z^{-1} \sin \Omega_0}{1-2z^{-1} \cos \Omega_0 + z^{-2}} = \frac{z \sin \Omega_0}{z^2-2z \cos \Omega_0 + 1}$	$ z > 1$
$\sigma[n] a^n \cos \Omega_0 n$	$\frac{1-z^{-1} a \cos \Omega_0}{1-2z^{-1} a \cos \Omega_0 + a^2 z^{-2}} = \frac{z(z-a \cos \Omega_0)}{z^2-2za \cos \Omega_0 + a^2}$	$ z > a $
$\sigma[n] a^n \sin \Omega_0 n$	$\frac{z^{-1} a \sin \Omega_0}{1-2z^{-1} a \cos \Omega_0 + a^2 z^{-2}} = \frac{za \sin \Omega_0}{z^2-2za \cos \Omega_0 + a^2}$	$ z > a $