Abstract
This paper presents the instantaneous frequency, the most important techniques in instantaneous frequency determination for nonstationary signals and some representatives’ simulations.

Key words: instantaneous frequency (IF), Wigner-Ville distribution, multicomponent signals.

I. INTRODUCTION
The instantaneous frequency of a signal is a parameter used in communications, seismic processing, radar signals and biomedical applications. The IF is a good descriptor of some physical phenomenon. The importance of the instantaneous frequency result from the fact that in many applications the signals is often nonstationary, a simple example being the chirp signal. In seismic processing these signals have the advantage that their spectral characteristics can be controlled including duration, bandwidth and energy. Another application is for estimation of Doppler frequency shift in radar returns. For all these signals, the IF is an important characteristic.

II. FREQUENCY ESTIMATION TECHNIQUES
The signal model which has often used is:
\[ z(n) = Ae^{i2\pi fn} + \varepsilon(n) \]
where \( A \) is the amplitude, \( f \) is the frequency, \( z(n) \) is the discrete complex observation sequence and \( \varepsilon(n) \) is the complex white Gaussian noise sequence [1].

The estimate for the frequency of a single sinusoid in white Gaussian noise has been shown to be given by finding that frequency at which the spectrum attains its maximum. This may be implemented with an initial coarse search on the bins of a Fast Fourier Transform (FFT) and a subsequent interpolation procedure. As long as the coarse frequency estimate falls within the main lobe of the frequency response, this technique converges to the correct global maximum. This estimate meets the Cramer-Rao bound (CRB) above a SNR threshold, the bound being given by:

\[ \text{disp}\left[\hat{f}\right] \geq \frac{12}{(2\pi)^2 \left(A^2 / \sigma^2\right) N \left(N^2 - 1\right)} \]

where \( N \) is the number of independent samples in data, \( A \) is the signal amplitude and \( 2\sigma^2 \) is the complex noise variance.

The estimated variance departs quite dramatically from the CR bound once the SNR falls below a threshold value, a phenomenon that is common in nonlinear estimators.

This method can be computationally too intensive in some applications. In an attempt to find frequency estimators, which reduce computation, many researchers have turned to parametric methods. These methods model the signal as having a rational transfer function. It is often computationally advantageous to assume that the numerator of the transfer function is a constant. Such models are said to be auto-regressive (AR), or linear predictive.

Tretter introduced another frequency estimation technique [8]. He showed that for a complex sinusoid in white Gaussian noise at high SNR, the phase might be approximated well as a linear function of time, imbedded in an additive white Gaussian noise process. He used a linear regression technique to estimate the frequency. One problem with Tretter’s algorithm is that the first stage of the algorithm necessitates extracting the phase from data. This can be the source to significant errors. Kay obtained a modified form of this estimator by fitting a model to adjacent phase difference estimates, rather than the phase values themselves, thus avoiding the phase unwrapping problem. The resulting estimator is simply a smoothing of phase differences with a quadric window.

The methods described in this section have dealt specifically with sinusoidal frequency estimation. They provide a good basis to understand the more complicated problem of estimating time-varying frequencies.

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III. INSTANTANEOUS FREQUENCY ESTIMATION TECHNIQUES

Definition of the Discrete-Time IF

The definition for the IF of a real continuous-time signal \( s(t) \), was given by Ville as [1]:

\[
\Phi(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{d}{dt} \arg\{S(f)\} \, df
\]

where \( \Phi(t) \) is the phase of the analytic signal associated with \( s(t) \).

To implement discrete-time IF estimators, one must first know how the differentiation operation may be realized in discrete time. One solution is to use a discrete finite impulse response (FIR) differentiator. The discrete-time IF may then be defined as:

\[
f_i(n) = \frac{1}{2\pi} \Phi(n) \ast d(n)
\]

where \( d(n) \) is the impulse response of an FIR differentiating filter and \( \ast \) denotes convolution in time.

Such filters have practical problems, however, since they exaggerate the effects of high frequency noise. Good approximations to the differentiation operation in discrete-time can be obtained by using a phase differencing operation. This approach is very computationally efficient and in general yields better noise performance than is obtainable with (2).

The forward and backward finite differences (FFD) and (BFD) defined by (3) and (4) are two commonly used phase differencing operations:

\[
\hat{f}_f(n) = \frac{1}{2\pi} \left[ \Phi(n+1) - \Phi(n) \right]
\]

\[
\hat{f}_b(n) = \frac{1}{2\pi} \left[ \Phi(n) - \Phi(n-1) \right]
\]

We can also estimate the discrete-time IF using central finite difference (CFD):

\[
\hat{f}_c(n) = \frac{1}{2\pi} \frac{\Phi(n+1) - \Phi(n-1)}{2}
\]

\[
= \frac{1}{4\pi} \left[ \Phi(n+1) - \Phi(n-1) \right]
\]

Of the three discrete IF estimators in (3), (4) and (5), the one defined by (5) has some advantages. Firstly it is unbiased and has zero group delay for linear FM signals and secondly it corresponds to the first moment in frequency of a number of TFD’s.

We can define a class of phase difference estimators, which are unbiased for polynomial phases of arbitrary order. For phase given by:

\[
\Phi(n) = \sum_{i=0}^{p} a_i n^i
\]

the instantaneous frequency is obtained as:

\[
\hat{f}_i(n) = \frac{1}{2\pi} \sum_{i=1}^{p} ia_i n^{i-1}
\]

The Relation Between IF and Time-Frequency Distributions

Time-frequency distributions TFD were introduced as a means of representing signals whose frequency content is varying with time and for which both time domain representations and frequency domain representations are inadequate to describe the signal appropriately.

Ideally, one would expect from a time-frequency representation a signal that peaks about the IF, with a spread related to the FT of the envelope of the signal [3].

For monocomponent signals of the form of \( a(t) \cdot e^{j\phi(t)} \) it would be intuitively satisfying to generate a TFD of the following form:

\[
TF(t, f) = A(t, f) \ast \delta(f - f_i(t))
\]

where \( A(t, f) \) is the time-frequency representation of the amplitude function, \( a(t) \). Thus the amplitude and phase would be separable, providing an easily interpretable distribution. The distribution is centered on the time varying IF with the amplitude information distributed in time and frequency.

It would be an advantageous feature of a distribution to allow estimation of the IF simply by peak detection. With signals whose phase functions are quadric (i.e., 3rd order derivative zero), the Wigner-Ville distribution of the signal will be of the form of (8) and IF estimation can be achieved by peak detection. We can also observe the spread about the IF at each instant time.

IF Estimation Based on the Moments of TFD’s

Cohen has formulated a class of two-dimensional functions (TFD’s), which may be used to represent the distribution of signal energy in time and frequency [9].

The discrete time expressions for these functions were given by:
\[
\rho(n,k) = \sum_{m=-M}^{M} \sum_{p=-m}^{m} G(p-n,m)z(p+m) \\
\cdot z^*(p-m)e^{-j\pi mk/N}
\]

(9)

where \(G(n,k)\) is a window function which selects a particular TFD, and \(M = (N - 1)/2\).

A number of TFD’s (e.g., the Wigner-Ville Distribution WVD) yield the IF through their first moment and many other TFD’s (e.g., the Short-Time Fourier transform) yield approximations to the IF through their first moment.

TFD first moments, provide another means of estimating the IF.

White and Boashash considered the problem of estimating the IF of a Gaussian random process using WVD first moments [6], [10].

The useful aspect of estimating the IF in this manner is that masking or other forms of preprocessing in time-frequency plane can be performed so as to reduce noise effects or to estimate IF laws of the various components separately.

The masking or time-varying filtering operation has resulted in a significant variance reduction. The IF derived the first moment of the discrete WVD is given by:

\[
\hat{f}_c(n) = \frac{M}{2\pi} \left\{ \sum_{k=0}^{M-1} e^{\frac{2\pi k}{M}} W^g(n,k) \right\} \mod 2\pi
\]

(10)

where \(W^g(n,k)\) is the discrete Wigner-Ville distribution defined by:

\[
W^g(n,k) = \sum_{m=-M}^{M} z\left(n + \frac{m}{2}\right)z^*\left(n - \frac{m}{2}\right)e^{-j\pi mk/N}
\]

(11)

Because this method is computationally demanding (requiring calculation of a TFD, masking and first moment calculations) and is not generally statistically optimal, other methods are preferred.

The IF, or an approximation of it may similar be obtained from moments of the discrete time TFD’s defined in relation (9). Such moments can be shown to be approximately a smoothed CFD estimator. That is:

\[
m^\phi_p(n) \cong \left\{ \hat{f}_c(n) * G(n,1) \right\} \mod \left( \frac{f_c}{2} \right)
\]

(12)

\(\hat{f}_c(n)\) is as specified in relation (5) and \(G(n,1)\) is the function from relation (9), \(G(n,k)\) \(\mod 2\).

In practice, this method is computationally demanding, since it requires calculation of a TFD, a masking operation and then an inversion procedure.

Because of its equivalence with a smoothed CFD operation, it is used only in very specific applications.

**IF Estimation Based on the Peak of TFD’s**

**Peak of the WVD**

The WVD peak was proposed as an IF estimation technique and it was applied to determining absorption and dispersion parameters in seismic processing.

The utility of WVD is its ability to localize energy along the IF law. If the signal under consideration has a linear frequency law and constant amplitude, the WVD will reduce to a row of delta functions on the IF law. From this situation we can estimate the IF [4].

The existence of any nonlinearity in time-varying spectral estimation [11]. It also tends to suppress cross terms so that it is suited to multicomponent signals. The signal-dependent TFD’s of Baraniuk and Jones and the adaptive techniques for high-resolution time-varying spectral estimation of Fineberg and Mammoone also are interesting.

**Other TFD’s**

The Zhao-Atlas-Marks Distribution seems to have good time-frequency localization and good noise performance, making it a useful prospect for IF estimation [11]. It also tends to suppress cross terms so that it is suited to multicomponent signals. The signal-dependent TFD’s of Baraniuk and Jones and the adaptive techniques for high-resolution time-varying spectral estimation of Fineberg and Mammoone also are interesting.

**IF Estimation for Multicomponent Signals**

The linear time-frequency representations have the advantage of a better time-frequency localization. The bilinear time-frequency representations have the advantage of a better energy’s concentration on the curve of instantaneous frequency. This kind of time-frequency representations contains interference terms. This is the reason why it is more difficult to interpret the image of such a time-frequency distribution [2].

A new method to reduce these interference terms is presented in [5]. This method contains the followings steps:

1. The computation of the Gabor time-frequency representation;
2. The Gabor representation’s nonlinear filtering; it results a prototype time-frequency representation;
3. The computation of the bilinear Wigner-Ville representation;
4. The multiplication of prototype time-frequency representation with the Wigner-Ville representation;

IV. SIMULATION RESULTS

First signal is a monocomponent signal with linear FM (chirp). The frequency domain is 0.1 Hz to 0.3 Hz, represented by 128 points. The ideal IF is represented in figure 1.

The SNR is 0 dB. Figures 2 and 3 show linear FM IF estimates obtained from the WVD first moment obtained with and without a preliminary masking operation. Figure 3 contains a better result than figure 2.

In figure 4 there are the WVD discrete peaks. It can be observed a good localization on IF representation but it contains interference terms.

The second signal analyzed is composed by two chirp signals, one with an increasing in time instantaneous frequency and the other with a decreasing in time instantaneous frequency and a cosine signal. The new interference terms’ reduction method was tested for an input SNR of 0.3 dB. The results obtained are presented in figure 5. The presence of the additive noise is very visible on the image of the Gabor time-frequency representation (figure 5a). The prototype time-frequency representation is also perturbed (figure 5b). In figure 5c) is WVD and it can be observed that there are two categories of interference terms: the first contains the terms situated in the vicinity of the three components and the second contains the terms situated at a large distance. The interference terms from the second class are produced by the noise component. Figure 5d) presents the modulus of the new time-frequency representation. It can see that a lot of interference terms are eliminated and there is a good estimation of IF.
V. CONCLUSIONS

In this paper there are the necessary concepts to obtain and interpret the instantaneous frequency. The relation between IF and time-frequency distributions is given. There are some techniques of IF estimation. New improvements in IF estimation for multicomponent signals will be in the next material.

REFERENCES


