

# Time-Frequency Methods for Multicomponents Signals

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**Abstract**— Instantaneous frequency (IF) is a very important parameter in a large number of applications. Generally, the IF is a non-linear function of time. For such cases the analysis of time-frequency content provides an efficient solution. This paper is a comparative study of the performances in IF estimation of the two time-frequency based methods. Limitations of the two methods are presented and some possible improvements. The first estimation method uses the complex argument distribution (CTD) and the second one uses the ridges extraction method from the time-frequency distribution based on mathematical morphology operators (TF-MO). Multicomponent signals with non-linear IF corrupted by Gaussian white noise are considered as numerical examples.

## I. INTRODUCTION

In signal processing the decision (detection, denoising, estimation, recognition or classification) is a basic problem. Knowing that the real environments are generally highly non-stationary, it is necessary to use a method able to provide suggestive information about the signal structure. A potential solution is based on time-frequency representations that provide a good concentration around the law of the IF and realize a diffusion of the perturbation noise in the time-frequency plane.

The CTD has been introduced in [1] as an efficient way to produce almost completely concentrated representations along the IF, it considerably reduces the artifacts due to the complexity of the analyzed signal.

The TF-MO estimation method [2] is based on the conjoint use of two very modern theories, that of time-frequency distributions and that of mathematical morphology. This strategy permits the enhancement of the set of signal processing methods with the aid of some methods developed in the context of image processing.

The paper is organized as follow. In section II is presented the CTD. The TF-MO method is illustrated in section III. In section IV some simulation results are depicted. Section V will close this communication.

## II. COMPLEX TIME DISTRIBUTION

The complex time distribution as an IF estimator have been proposed and analyzed in [1]. It provides a highly concentrated distribution along the IF law with reduced interferences (noise and cross-terms).

Mathematically, the complex time distribution (CTD) of signal is defined as:

$$CTD(t, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} s\left(t + \frac{\tau}{4}\right) s^*\left(t - \frac{\tau}{4}\right) \times s^{-j}\left(t + j\frac{\tau}{4}\right) s^j\left(t - j\frac{\tau}{4}\right) e^{-j\omega\tau} d\tau \quad (2.1)$$

where the continuous form of the “complex-time signal” is:

$$s(t + j\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(j\omega) e^{-\omega\tau} e^{j\omega t} d\omega, \quad (2.2)$$

where  $S(j\omega)$  is the Fourier transform of signal  $s$ .

The main property of the CTD consists in the capability to attenuate the high-order terms of the polynomial decomposition of the IF, for signals of the form  $s(t) = r e^{j\phi(t)}$ . The spread factor  $Q(t, \tau)$  around the IF for this distribution is:

$$Q(t, \tau) = \phi^{(5)}(t) \frac{\tau^5}{4^4 5!} + \phi^{(9)}(t) \frac{\tau^9}{4^8 9!} + \dots \quad (2.3)$$

The complex time distribution can be considered to be a corrected Wigner-Ville distribution (WVD), where the corrected term is:

$$c(t, \tau) = s^{-j}\left(t + j\frac{\tau}{4}\right) s^j\left(t - j\frac{\tau}{4}\right). \quad (2.4)$$

As proved in [4], in the case of the CTD, this function has a fifth order dominant term (for comparison, the spectrogram and the Wigner-Ville distribution have a second and a third order dominant term, respectively), which corresponds to a drastic reduction of the higher terms of  $Q(t, \tau)$ .

The CTD satisfies some important properties:

1) the CTD is real for the frequency modulated signals  $s(t) = r e^{j\phi(t)}$ .

2) the time-marginal property  $\frac{1}{2\pi} \int_{-\infty}^{\infty} CTD(t, \omega) d\omega = |s(t)|^2$ .

3) the unbiased energy condition

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} CTD(t, \omega) dt d\omega = \int_{-\infty}^{\infty} |s(t)|^2 dt = E_s.$$

4) the frequency-marginal property.

5) time-frequency shift-invariance properties.

6) the CTD of the scaled signal  $\sqrt{|a|}s(at)$  is  $CTD(at, \omega/a)$ .

The discrete form of the CTD is:

$$CTD(n, k) = \sum_{m=-N/2}^{N/2-1} w(m)s(n+m)s^*(n-m) \times s^{-j}(n+jm)s^j(n-jm)e^{-j(2\pi/N)4mk} \quad (2.5)$$

where  $w(m)$  is a lag window and  $N$  is the signal duration.

For multicomponent signals the algorithm [1] for cross-terms reduced realization of the CTD is as follows:

1) Calculate the S-method distribution [6]

$$SM(n, k) = \sum_{l=-L}^L STFT_s(n, k+l)STFT_s^*(n, k-l), \quad (2.6)$$

where  $STFT_s(n, k)$  is the short-time Fourier transform of the signal  $s(n)$ .

2) Calculate the cross-terms reduced version of the correction component term for the  $CTD(n, k)$  in the following way:

a) Find the position  $k_1(n)$  of the  $STFT_s(n, k)$  maximum, for a given time instant  $n$

$$k_1(n) = \arg \left\{ \max_k STFT_s(n, k) \right\}. \quad (2.7)$$

b) In the region  $[k_1(n) - M_k, k_1(n) + M_k]$  which is considered as a signal component, calculate the correction term according to (2.10).

c) Put  $STFT_s(n, k) = 0$  in the region where the correction term was calculated, and repeat the steps from a) to c) for each signal component.

Form the correction signal as

$$C(n, k) = FT_m \left\{ \sum_{p=1}^P c_p(n, m) \right\}. \quad (2.8)$$

3) Convolve results from 1) and 2)

$$CTD(n, k) = \sum_{l=-L}^L SM(n, k+l)C(n, k-l). \quad (2.9)$$

The correction component term is obtain with

$$c_1(n, m) = w(m)e^{j \ln |s_1(n-jm)/s_1(n+jm)|} = w(m)e^{j(4\pi/N)mk_1(n)} \times \exp \left( j \ln \left| \frac{\sum_{k=-M_k}^{M_k} STFT_s(n, k+k_1(n))e^{j(2\pi/N)(n-jm)k}}{\sum_{k=-M_k}^{M_k} STFT_s(n, k+k_1(n))e^{j(2\pi/N)(n+jm)k}} \right| \right) \quad (2.10)$$

### III. TIME-FREQUENCY MORPHOLOGICAL OPERATORS METHOD

The estimation method based on time-frequency and image processing techniques has been introduced in [2]. The quality of estimating the IF depends on the time-frequency distribution and on its ridges projection mechanism. The TF-MO method proposes a time-frequency representation based on cooperation of linear and bilinear distributions: the Gabor and the Wigner-Ville distributions. It is known [3] that the Gabor representation has a good localization and free interference terms properties. Unfortunately, the linear distributions, except the Discrete Wavelet transform, correlate the zero mean white input noise, as shown in [4]. The WVD is a spectral-temporal density of energy that does not correlates the input noise, thus having a spreading effect of the noise power in the time-frequency plane [2]. The WVD has also a good time-frequency concentration.

To combine these useful advantages, the time-frequency distribution is calculated according to the following algorithm [2]:

- 1) Calculate the Gabor transform for the signal  $s$ ,  $G(t, \omega)$ .
- 2) Filter the image obtained with a hard-thresholding filter:

$$Y(t, \omega) = \begin{cases} 1, & \text{if } |G(t, \omega)| \geq tr \\ 0, & \text{if } |G(t, \omega)| < tr \end{cases} \quad (3.1)$$

where  $tr$  is the threshold used.

3) Calculate the WVD for the signal  $s$ ,  $WV(t, \omega)$ .

4) Multiply the modulus of the  $Y(t, \omega)$  distribution with the  $WV(t, \omega)$  distribution.

In step 2) the proposed threshold value is:

$$tr = \frac{\max_{(t, \omega)} \{ |G(t, \omega)| \}}{5} \quad (3.2)$$

This operation decreases the amount of noise that perturbs the ridges of  $G(t, \omega)$  and brings to zero the values in the rest of the time-frequency plane. The effect of the multiplication in step 4)

is the reduction of the interference terms of the WVD and the very good localization of the ridges of the resulting distribution.

To estimate the ridges of the obtained distribution, some mathematical morphology operators are used, the above resulting distribution being regarded as an image. This mechanism is applied through the following steps [2]:

- 1) Convert the image obtained in step 4) in the procedure described earlier in binary form.
- 2) Apply the dilation operator on the image in 1).
- 3) Skeletonization of the last image, an estimation of the IF of the signal being obtained. This image represents the result of the TF-MO method. The conversion in binary form realizes a denoising of the time-frequency distribution. The role of the dilation operator is to compensate the connectivity loss, produced by the preceding conversion. The skeleton produces the ridges estimation.

#### IV. RESULTS

In [8], [1] it has been showed that the CTD distribution and TF-MO method are very effective in IF estimation for the case of monocomponent signals. It has been observed that for monocomponent signals with highly non-linear IF and high SNR, the CTD distribution is more adapted than the TF-MO method as well as for signals with a time-frequency structure not so complicated, the performances are relatively the same.

Moreover, when the components are well separated in frequency, the algorithm for multicomponent signals in the CTD case provides good results.

##### A. Example 1

Consider a noisy multicomponent signal with non-linear IF with relatively closed in frequency components:

$$s(t) = \exp\{j(-2\pi t^3 + 5\pi t)\} + \exp\{j(-2\pi t^3 + 12\pi t)\} + n(t)$$

within the interval  $[-1, 1]$  and  $N=128$  where  $n(t)$  is a Gaussian white noise and SNR=30dB.

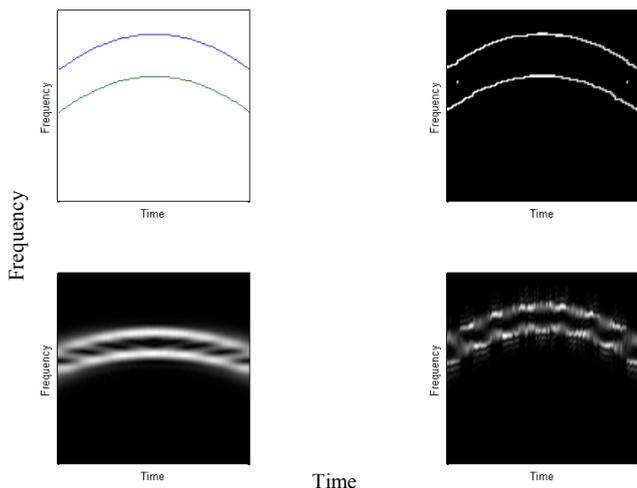


Figure 1. Ideal IF (first line and first column), TF-MO method (first line and second column), STFT (second line and first column), CTD (second line and second column) for SNR=30dB

From Fig.1 can be observed that in the TF-MO method, because the signal components are relatively closed in frequency, the cross-terms are not completely removed, thus the quality of the IF estimation can be affected.

In addition, because the STFT of the components overlap, the algorithm [1] for cross-terms reduced realization of the CTD, which assumed to calculate a correction term in the region where the signal component is located, is not very adapted. The CTD in Fig.1 illustrates that fact, where the bias and the variance are significant. Also, the performances of the TF-MO method and CTD distribution are both affected by the influence of the noise.

##### B. Example 2

Consider a noisy multicomponent signal with non-linear IF whose components intersect:

$$s(t) = \exp\{j(5\pi t^3 - 9.5\pi t)\} + \exp\{j(-7\pi t^3 + 5\pi t)\} + n(t)$$

within the interval  $[-1, 1]$  and  $N=128$  where  $n(t)$  is a Gaussian white noise and SNR=30dB.

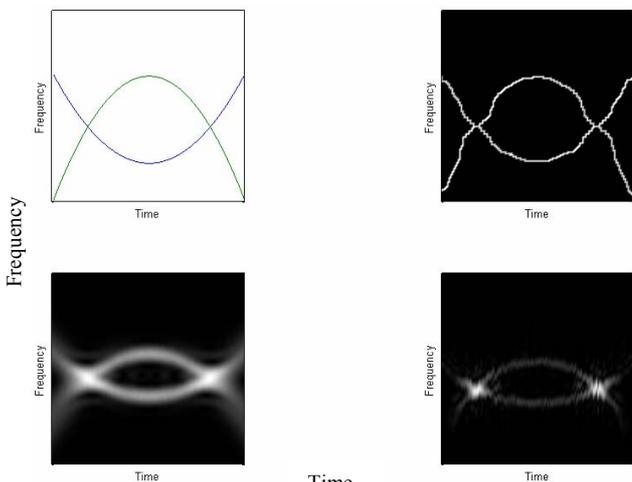


Figure 2. Ideal IF (first line and first column), TF-MO method (first line and second column), STFT (second line and first column), CTD (second line and second column) for SNR=30dB

It can be noticed that from Fig.2 in the TF-MO method, the variance is affected by the interference terms between the signal components and by the noise.

For the signals whose components intersects and are not the same widths, it has been observed that the algorithm in the CTD case is not effective, the better results are obtained when the time-frequency image is regarded as a single component and one correction signal is calculated.

##### C. Example 3

Consider a noisy multicomponent signal with non-linear IF whose components are closed in frequency:

$$s(t) = \exp\{j(5\pi t^2 - \pi t)\} + \exp\{j(3\pi t^2 + 7\pi t)\} + n(t)$$

within the interval  $[-1, 1]$  and  $N=128$  where  $n(t)$  is a Gaussian white noise and  $\text{SNR}=30\text{dB}$ .

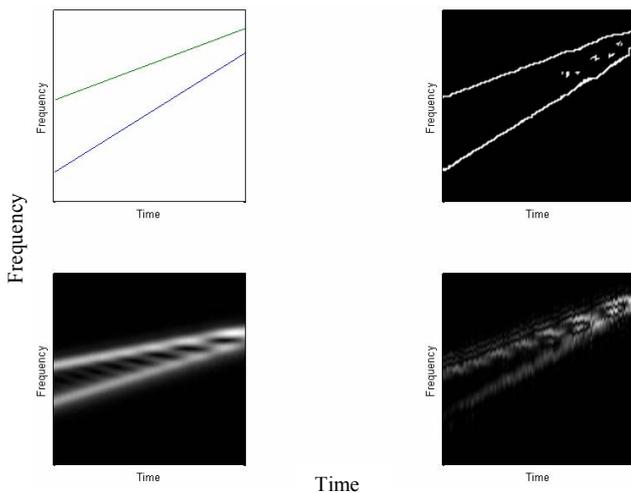


Figure 3. Ideal IF (first line and first column), TF-MO method (first line and second column), STFT (second line and first column), CTD (second line and second column) for  $\text{SNR}=30\text{dB}$

Fig.3 proved that in the region where the signal components are very closed in frequency, the interference terms are very important, and they can't be completely removed, thus degrading the performance of the estimation.

From the analysis of the examples considered, particularly when signal components are closed and intertwined in the time-frequency plane, the procedure [1] in the CTD, of determining the time-frequency region of the components, where the correction component term is calculated, has a great effect on the quality of the results. The auto-terms and noise interferences can induce false maximum detection of the  $STFT_s(n, k)$  which don't belong to the signal component. A possible solution could be a detection technique based on filter-bank processing.

From these numerical examples, it can be noticed that the TF-MO method is very dependent on the choice of the threshold  $tr$ . A high value can preserve the noise peaks in the time-frequency plane outside the region where the signal component is located and the interference terms between the signal components. This side effect is very significant for relatively high noise, thus degrading the estimation process. The performance can be improved by applying the morphological operators only in the region around the signal component. This can be done using a detection technique. Moreover, the parameters of the morphological operators have an important role for the ridges

extraction precision and it has been observed that for high SNR skipping the application of the dilation operator in the TF-MO method provides an improvement of the IF estimation.

A low value for the  $tr$  can cause connectivity breaks of the time-frequency distribution used, this inconvenient being compensated by the reconstruction capacity of the morphological operators.

## V. CONCLUSIONS

In this paper, it has been analyzed the performances and the limitations of the CTD distribution and TF-MO method, for multicomponent signals whose components are close and intertwined in the time-frequency plane.

The TF-MO method is more adapted for such signals. The performance can be improved in the algorithm [1] for CTD, by calculating the correction term, in the region determined by using a filter-bank processing.

Further research could be directed toward the estimation of the polynomial order of the estimated IF, the conception of a new IF estimation method combining the qualities of the two methods analyzed in this paper.

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