ECG Statistical Denoising in the Wavelet Domain

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Abstract – The paper presents a denoising algorithm particularly suited to ECG signals processing. The main stage of this algorithm consists in a MAP filtering in the wavelet domain. Its effectiveness relies on the qualities of the wavelet transform and of the statistical filter used. Tests made on ECG signals, in realistic conditions, showed very promising results. The noise is almost completely removed, while the useful waveforms are preserved.

Keywords: denoising, wavelet, ECG, MAP

I. INTRODUCTION

The clinical electrocardiogram (ECG) records the changing potentials of the electrical field generated by the heart. Electrocardiography can be used, within limits, to identify anatomical, metabolic, ionic and hemodynamic changes. Automatic ECG signal processing aims the detection and even the prevention of cardiac illness and can be very helpful for the cardiologists. Unfortunately, ECG signal acquisition process is subjected to various disturbing perturbations like power-line interferences, electromyogram noise caused and baseline drift. All these unwanted phenomena make the automatic interpretation of the signal a difficult and task. Under these conditions, a pre-treatment of the signal is highly desirable for removing such interferences. A part of this procedure will be next referred to as denoising. The removal of the baseline drift will be referred to as correction.

The wavelet transform (WT) has been extensively used in the signal processing community in order to highlight informative representations of non-stationary signals. WT is able to simultaneously provide time and frequency information and offers good temporal localization for high frequencies and high–frequency resolution for low frequencies. ECG record is a non-stationary signal, so WT-based denoising particularly matches to it. The architecture of a wavelet-based denoising system relies on WT ability to concentrate the useful signal energy into a small number of wavelet coefficients. The algorithm introduced by Donoho [1] uses discrete wavelet transform (DWT) and it has three steps:

1. DWT is applied on the noisy signal;

2. Wavelet coefficients are filtered (procedure which is sometimes referred to as "shrinkage" or "thresholding").

3. Remaining coefficients are back-converted in time domain to estimate the useful signal.

Generally, the results are highly dependent on the wavelet mother used (stages 1 and 3) and on the filtering procedure chosen (stage 2). Some modern wavelet denoising techniques implement a MAP filtering in stage 2 of the algorithm, taking into account the statistical properties of the wavelet coefficients. Such a method, which is used for processing the ECG signal in noisy conditions [2], adapts the wavelet domain empirical Wiener filter presented in [3] to the particular case of ECG signals. The statistical properties of the wavelet coefficients are estimated through a pilot signal. The pilot is obtained by applying the classical Donoho's algorithm on the input noisy signal. Next, a MAP filtering in wavelet domain is performed, using the parameters estimated through the pilot. The mother wavelets, used in the two stages (pilot estimation and MAP filtering) are different. Using mother wavelets with compact temporal support in the first stage allows an accurate preservation of the areas around the QRS complex [2]. On the other hand, the use of mother wavelets with good frequency localization in the second stage of the algorithm refines the shapes of P and T waves. The analytical solution required for implementing this kind of filter uses the hypothesis that both useful and noise samples (wavelet coefficients when the filter is applied in WT domain) have Gaussian probability density functions (pdf). The two most important features of such a filtering technique are: realistic a-priori assumptions regarding statistical properties of both signal and noise components and a good estimation of the parameters that describe these properties.

In this paper we propose a denoising method with a single stage associating the stationary wavelet transform (SWT) with a MAP filter, referred to as "bishrink" [4]. This filter relies on an improved method to estimate the statistical parameters of the wavelet coefficients. Thus, the interscale dependence of wavelet coefficients is taken into account when solving the MAP filter equation. In opposition with the zero order Wiener filter which is constructed on the basis of one dimension probability density functions (pdfs), the bishrink filter is built using bivariate pdfs, which take into account the interscale dependence of wavelet coefficients. The two bivariate pdfs used for the construction of the bishrink filter are well adapted to the characteristic shape of ECG signal. In ECG denoising, exact preservation of the useful waveforms is critical. However, distortions are sometimes introduced in the useful signal by wavelet domain denoising. The distortions can be controlled by a proper selection of the WT and of its features: the mother wavelets and the number of decomposition levels, K. In the present study we have selected the mother wavelets with the best time-frequency localization from the Daubechies family [5] and a number of K=8 decomposition levels [6]. As

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illustrated in [3], the use of mother wavelets with good temporal localization is recommended in denoising applications. This choice mitigates pseudo-Gibbs phenomenon effects, usually associated with the shrinkage of the DWT coefficients. Thus, a good preservation of the zones around QRS is provided this way. But the ECGs contain smooth regions with low frequency content. Their treatment require mother wavelets with good frequency localization. The value *K* must be selected in accordance with the sampling frequency used for the acquisition of the ECG, f_s. The ECG is a quasi-periodic signal, whose periodicity is given by the heart beats rate. The fundamental period is noted *T* and can be determined by measuring the pulse of the patient: $(T \in [0.75,\ 1]s)$. The time resolution of the Kth decomposition level must be higher or at least equal to T. The value of K must be selected to satisfy the condition:

$$2^K / f_s \geq 1 \quad (1)$$

As shown in [6], for a the sampling frequency of 360 Hz the last condition becomes *K=8*. The theoretical background of our algorithm especially considers the suppression of wide band EMG noise, but good practical results are provided for the power-line interference too. The baseline wandering of the denoised ECG can be also corrected applying the method proposed in [6]. In section II, the proposed denoising algorithm is presented. Next, simulation results are shown. Section IV contains a few concluding ideas and draws future possible directions to continue our work on this subject.

II. METHOD

The architecture of the proposed denoising system is presented in fig. 1. To the input we get the useful signal (s) additively perturbed by noise (p):

$$x = s + p \quad (2)$$

Figure 1. The proposed algorithm.

An example of input waveform is presented in figure 2. The baseline wandering can be observed in Figure 2 a) and the noise can be observed in figure 2 b). It is more visible on the smooth regions of the ECG, which correspond to the P and T waves. We have selected the Stationary Wavelet Transform (SWT) despite its high redundancy due to its perfect translations invariance [6]. The classical denoising method proposed by Donoho [1] is applied in the SWT domain. Thus, the signal is converted in the wavelet domain, the resulting wavelet coefficients are filtered ("bishrink" block, fig. 1) and then back-converted in time domain, with the aid of the Inverse Stationary Wavelet Transform (ISWT). This operation takes into account the additive nature of the noise, illustrated by equation (2). The bishrink is a MAP filter[4]. In order to provide robustness and superior performance to our algorithm,

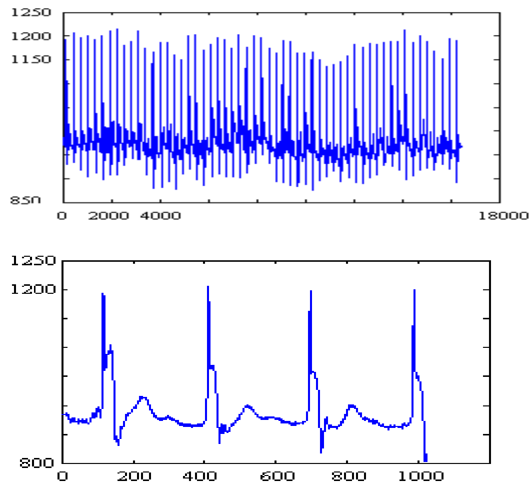


Figure 2. The waveform of the input signal: a) the whole waveform, b) first three beats.

we made realistic a-priori assumptions regarding pdfs of the useful and noise coefficients. To the output of the SWT block, we get a sequence of wavelet coefficients, as follows:

$$\mathbf{w} = \mathbf{u} + \mathbf{n}, \quad (3)$$

where: $\mathbf{w} = (w_1, w_2)$; $\mathbf{u} = (u_1, u_2)$ and $\mathbf{n} = (n_1, n_2)$ and ^1u and ^1n denotes the useful and the noise coefficients respectively and ^2u and ^2n denotes their parents (wavelet coefficients having the same coordinates but located at the next decomposition level). Thus, we have considered the parent coefficients to take into account the intra-scale dependence.

Using Bayesian rules, the MAP estimation of ^1u can be computed as:

$$\hat{\mathbf{u}}(\mathbf{w}) = \arg\max_{\mathbf{u}} \left(\log(p_{\mathbf{w}/\mathbf{u}}(\mathbf{w}/\mathbf{u}) \cdot p_{\mathbf{u}}(\mathbf{u})) \right) = $$
$$= \arg\max_{\mathbf{u}} \left(\log(p_{\mathbf{n}}(\mathbf{w} - \mathbf{u})) + \log(p_{\mathbf{u}}(\mathbf{u})) \right) \quad (4)$$

The bishrink was considered a bivariate Gaussian distribution for the noise coefficients, with zero mean and variance σ_n^2:

$$p_{\mathbf{n}}(\mathbf{n}) = \frac{1}{2\pi\sigma_n^2} \cdot e^{-\frac{n_1^2 + n_2^2}{2\sigma_n^2}}. \quad (5)$$

This assumption makes possible to find an explicit solution of equation (4). We have no a-priori information about the noise which affects the ECGs from the chosen database which could be used to verify assumption (5). For the useful signal coefficients pdf, a bivariate Laplacian distribution seems to be well suited to the characteristic shape of the ECG signal. This supposition is supported by empirical work on large ECG databases [7]. In fact, the WT of an ECG signal consists of a small number of high value wavelet coefficients (the limits of the electrical activity zones) and a large number of small value coefficients (the slow-evolution portions of the ECG). A heavy-tailed distribution for these coefficients seems therefore far more realistic than a Gaussian-one, and the particular case of a Laplacian probability density function (pdf) becomes attractive by its computational tractability. Consequently, we take [4]:

$$p_{\mathbf{u}}(\mathbf{u}) = \frac{3}{2\pi\sigma^2} \cdot e^{-\frac{\sqrt{3}}{\sigma}\sqrt{u_1^2+u_2^2}} \tag{6}$$

Under the considered assumptions, the solution of (3) is [4]:

$$\hat{u}_1 = \frac{\left(\sqrt{w_1^2+w_2^2} - \frac{\sqrt{3}\sigma_n^2}{\sigma}\right)_+}{\sqrt{w_1^2+w_2^2}} w_1 \tag{7}$$

where:

$$X_+ = \begin{cases} X, & \text{if } X > 0 \\ 0, & \text{if not} \end{cases}$$

This solution represents a soft thresholding filtering of the sequence of noisy observations with the optimal threshold value *Th*. This value is computed using the estimated standard deviations of the clean and noise coefficients:

$$Th(j,k) = \frac{\sqrt{3}\,\hat{\sigma}_n^2}{\hat{\sigma}(j,k)} \tag{8}$$

It is highly recommendable,that the threshold value is individually estimated for each coefficient $w(j,k)$ since $\hat{\sigma}$ (estimated standard deviation of the useful coefficients) must be estimated locally, in order to accurately track the sharp zones that exist in the signal. This parameter is separately estimated for each coefficient, using a sliding window:

$$\hat{\sigma}(j,k) = \sqrt{\left(\frac{\sum_i |w(j,i)|^2}{\upsilon} - \sigma_n^2\right)}, \tag{9}$$

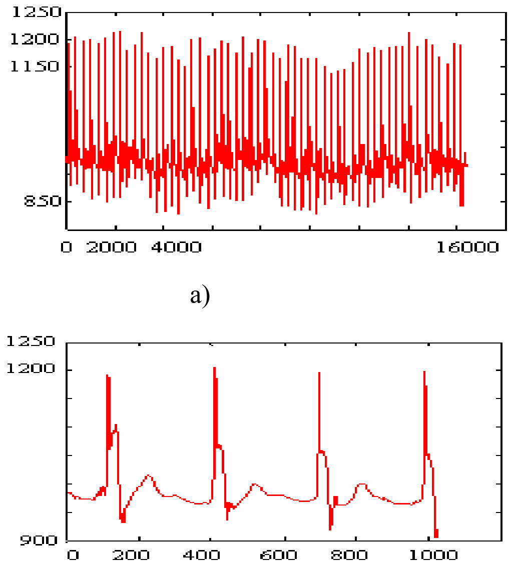
$$i = k - \frac{\upsilon - 1}{2}, \ldots, k + \frac{\upsilon - 1}{2}$$

where $w(j,i)$ represents the wavelet coefficient of the acquired signal, j standing for the decomposition scale and i for the position within the scale. ν is the length of the sliding window. Experimental work showed that the value ν=3 provides similar results with higher window lengths (ν=11 or 21). We have chosen for the experiments associated at this paper the value ν=11. On the other hand, the noise variance σ_n^2 is separately estimated for the first decomposition level j=1, using the detail wavelet coefficients $w(1,j)$ at that level:

$$\sigma_n^2 = \frac{\text{median}\left(\left|w(1,i)\right|\right)}{0.6745} \tag{10}$$

III. RESULTS

ECG test signals were chosen from MIT-BIH database. The sampling frequency of these signals is of 360 Hz, with a resolution of 11 bits/sample. In order to test our algorithm's effectiveness in real conditions, we applied it on a high number of ECG signals extracted from the MIT-BIH database. The result of its treatment is presented in figure 3. The effect of denoising can be appreciated comparing figures 2 b) and 3 b). For a better comparison, we superposed these waveforms in figure 4. By analyzing this figure it can be observed that the noise is almost completely suppressed without distorting the useful component of the input signal. Comparing figures 2 a) and 3 a) it can be observed that the proposed algorithm does not reduce the wander of the baseline, so we apply the baseline's correction strategy proposed in [6].



b)

Figure 3. The results obtained by the proposed denoising method for the processing of signals from figure 2, a) entire, b) first three beats.

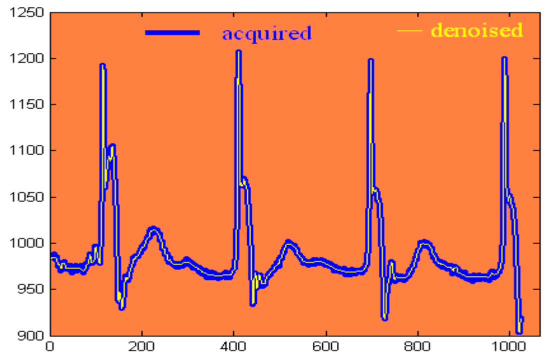
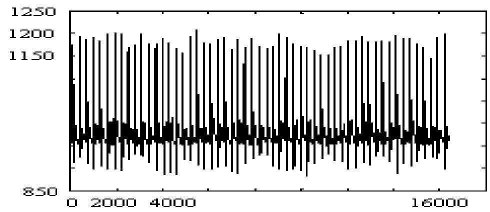
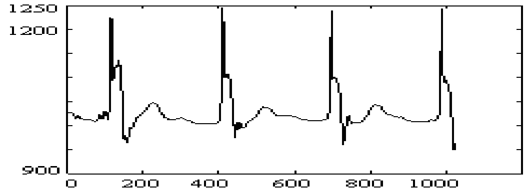


Figure 4. The superposition of the waveforms in figures 2 b) and 3 b)

The result obtained for the processing of the signal in figure 2 is presented in figure 5.



a.)



b.)

Figure 5. The result of the combination of the proposed denoising method with the baseline's wander compensation method proposed in [6], a) the whole waveform, b) first three beats.

In figure 6 the waveforms from figures 2 b), 3 b) and 5 b) are superposed, in order to better appreciate the effect of association of the proposed denoising method with the baseline's correction method proposed in [6]. On some test signals, the parasite component of 50 Hz can be clearly highlighted. Our algorithm has good practical results in these situations too. Computing its SWT and keeping only the detail coefficients we obtain the histogram represented in figure7.

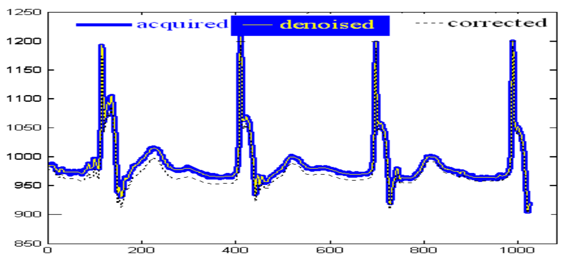


Figure 6. The superposition of waveforms in figure 2 b), 3 b) and 5 b) highlights the performance of the association of the two methods.

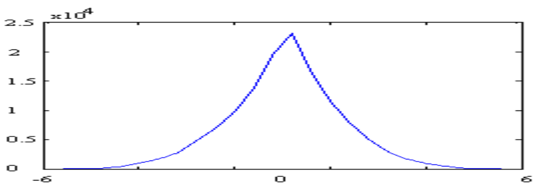


Figure 7. The histogram of the detail coefficients of the noise rejected by the proposed denoising algorithm.

It does not look exactly as the histogram of a Gaussian noise but it could still satisfy the equation (5).Thus, equation (5) refers to a bivariate pdf whereas the plot in figure 7 represents only a marginal distribution which does not take into account the interscale dependence of the wavelet coefficients. The bivariate pdf can be expressed as a product of two marginal distributions only if these one dimensional distributions refer to independent random variables. But, in our case, a inherent assumption is that the child coefficients are not independent versus their parents.

IV. CONCLUSIONS

A new ECG denoising algorithm is presented in this paper. The algorithm exploits the translations invariance of the SWT. This transform is computed using mother wavelets with very good time-frequency localization, the Daubechies' mother wavelets with two vanishing moments. We have selected a number of eight decomposition levels according to the considerations made in [6]. The proposed algorithm applies the bishrink filter in the SWT domain. This is a powerful MAP filter which takes into account the interscale dependence of the wavelet coefficients. Its parameters are chosen by making realistic assumptions on the statistical properties of the useful wavelet coefficients. Tests on several signals acquired from the MIT-BIH database were performed, in order to demonstrate the effectiveness of our algorithm. These ECGs represent signals affected by real noise. Further improvements are still possible. In the future, we will focus on deeper statistical study of the ECG wavelet coefficients to explain the form of the histogram in figure 7. If this noise is not white and Gaussian [8] then its variance must be estimated with a different equation. Another future research direction of our team will be the exploitation of inter-beat and intra-beat correlations of ECG signals.

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